no.3

LIBRARY
FISH & WILDLIFE SERVICE
PACIFIC OCEANIC FISHERY
INVESTIGATIONS

SEARS FOUNDATION FOR MARINE RESEARCH
BINGHAM OCEANOGRAPHIC LABORATORY, YALE UNIVERSITY

FEB 12 1952

JOURNAL OF MARINE RESEARCH

VOLUME X, 1951

NUMBER 3

ISSUED DECEMBER 31, 1951, NEW HAVEN, CONN., U. S. A.

JOURNAL OF MARINE RESEARCH

PUBLISHED BY

SEARS FOUNDATION FOR MARINE RESEARCH

BINGHAM OCEANOGRAPHIC LABORATORY, YALE UNIVERSITY

BOARD OF EDITORS

Chairman: A. E. PARR
American Museum of Natural History, New York

C. O'D. ISELIN
Woods Hole Oceanographic Institution

H. U. SVERDRUP
Norsk Polarinstitutt, Oslo, Norway

DANIEL MERRIMAN
Bingham Oceanographic Laboratory

C.-G. ROSSBY
Institut für Meteorologi, Stockholm, Sweden

THOMAS G. THOMPSON
Oceanographic Laboratories, University of Washington

MANAGING EDITOR

YNGVE H. OLSEN
Bingham Oceanographic Laboratory
Yale University
New Haven, Conn.

The Journal is normally issued in three numbers per year constituting one volume, except for volume V, published in 1942–1944, and volume VI, published in 1946 and 1947. All the income derived from subscriptions is used solely to increase the volume and improve the quality of the Journal for the benefit of its subscribers and contributors.

Subscription price three dollars per volume, or one dollar per number. Sverdrup Anniversary Number (Vol. VII, No. 3) available only by purchase of complete Vol. VII at $3.00. Address all communications to the Managing Editor. See inside rear cover for Note to Authors.

SEARS FOUNDATION FOR MARINE RESEARCH
BINGHAM OCEANOGRAPHIC LABORATORY, YALE UNIVERSITY

JOURNAL OF
MARINE RESEARCH

VOLUME X 1951 NUMBER 3

PARAMETERS OF TURBULENCE IN THE SEA

By

GORDON A. RILEY

Bingham Oceanographic Laboratory
and
Woods Hole Oceanographic Institution[1]

ABSTRACT

Dimensional analysis, combined with statistical examination of data, leads to the development of an equation for eddy diffusivity that is expressed in terms of readily measurable physical parameters—mean current speed, the distance between points of observation, the density of the water, and the dynamic molecular viscosity. The equation is applicable to both vertical and horizontal diffusion in water of indifferent stability. A further development which incorporates a stability term makes it applicable also to horizontal eddy diffusion in stable water. Study of problems of vertical diffusivity and conductivity in stable water fails to reveal a satisfactory method of utilizing the basic equation. An empirical relationship of the vertical coefficients with mean current speed and stability is described and discussed from the standpoint of its possible physical meaning. The character of the relationship leaves little doubt that vertical diffusivity and conductivity in stable water are fundamentally different from the other varieties of turbulent exchanges that have been examined.

INTRODUCTION

In recent years the writer and his associates (Riley, Stommel, and Bumpus, 1949; Riley, 1951; and Riley, 1952) have computed coefficients of eddy diffusivity and conductivity in a variety of oceanographic situations. Comparison of these coefficients with their physical environment reveals relationships that warrant further examina-

[1] Contribution No. 577 from the Woods Hole Oceanographic Institution.

tion. Data in the second and third papers cited indicate that both vertical and horizontal coefficients are proportional, other things being equal, to the square of the mean current speed. Since it is recognized that turbulence must be generated by shearing stress, this result possibly means that the mean square of the deviation in velocity in the sea[2] is proportional to the square of the mean velocity within wide limits.

In carrying the analysis further, it is therefore proposed as a first assumption that the average coefficient of eddy diffusivity A between two points x distance apart on any one of the three coordinate axes is a function of the square of the mean current speed v between the points. Second, certain other readily determinable quantities are proposed for inclusion in the examination, namely x, the density ρ, the dynamic molecular viscosity μ, and the stability of the water E.

EDDY COEFFICIENTS IN WATER OF INDIFFERENT STABILITY

When the stability factor is omitted, and when the current speed is required to appear as v^2, dimensional analysis shows that the remainder of the factors that have been listed are necessary for a satisfactory expression for eddy diffusivity, and the equation is

$$A = \frac{\alpha\,(\rho v x)^2}{\mu}, \tag{1}$$

where α is a dimensionless proportionality constant. It is perhaps worth mentioning that upon dividing through by μ, equation (1) becomes

$$\frac{A}{\mu} = \alpha R^2,$$

where R is the Reynolds number for the distance x.

In the absence of theoretical understanding or of assurance that all of the significant physical parameters are included in the equation, its validity can be tested only by empirical methods. A in the Appendix lists calculated eddy coefficients from the papers cited above, together with other necessary information. These coefficients are plotted in Fig. 1A against values computed from equation (1), using a value of 6×10^{-12} for α. This is statistically a good fit for the horizontal eddy coefficients. A larger value would be better for the vertical coefficients, and some of the estimates are in error by nearly an order of

[2] The postulated relationship apparently does not hold precisely in small-scale hydrodynamic experiments; for example, see McEwen (1950: 41).

magnitude. However, the results verify the essential correctness of the application of the terms v and x, both of which vary by several orders of magnitude in the appended data.

COEFFICIENTS OF LATERAL EDDY DIFFUSIVITY IN STABLE WATERS

Parr (1936) postulated that lateral[3] eddy diffusion is increased by stability and suggested that this is essentially a large-scale phenomenon. Both of these hypotheses are supported by the present investigation. Examination of the data on mid-depths in the Atlantic Ocean (B in APPENDIX) shows that the eddy coefficients are larger than those indicated by values computed from equation (1). However, evidence of a stability influence is lacking in Long Island Sound and is only slightly apparent in the data for the California offing, even though the stability exceeds the values for Atlantic Ocean mid-depths. The only obvious explanation of the apparent differences in the stability influence is that the latter may increase with distance x.

It is therefore assumed as a working hypothesis that the coefficient of lateral eddy diffusivity increases with stability and with distance. This relationship may be expressed in several forms that are dimensionally correct. A version that is empirically satisfactory except for one aberrant point is

$$A = \frac{\alpha(\rho v x)^2}{\mu}\left(1 + \frac{3\alpha g E x^2}{v^2}\right), \qquad (2)$$

where g is the acceleration of gravity. Fig. 1B compares the coefficients in B of the APPENDIX with values computed from equation (2).

VERTICAL EDDY COEFFICIENTS IN STABLE WATERS

Examination of coefficients of vertical eddy diffusivity in mid-depths of the Atlantic Ocean (C in APPENDIX) shows that equation (1) is applicable within reasonable limits of error. These coefficients were determined as average values between successive and equal intervals of σ_t. Therefore, the method of calculation incorporates a stability effect.

The examination of coefficients of vertical eddy conductivity is hampered by the fact that their computation did not involve a distance factor (cf. APPENDIX). This might not be a serious deterrent if, as indicated by the mid-depth eddy diffusivities, x in the equation might be replaced by a mixing length determined from some of the

[3] In Parr's discussion and in B of the APPENDIX, this term refers to quasihorizontal diffusion along surfaces of equal density.

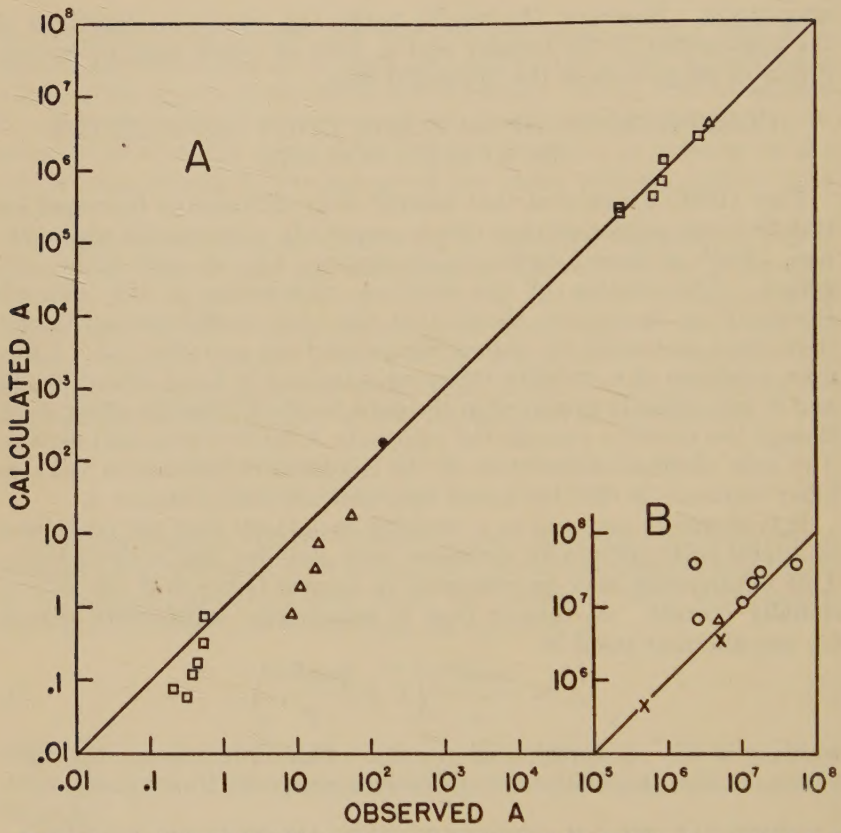

Figure 1. *A.* "Observed" eddy coefficients in water of slight stability (i. e., values in A of the APPENDIX derived from observed distributions of temperature or salinity) plotted against coefficients computed from equation (1). A logarithmic scale is used in order to show the entire range of values. *B.* Comparison of "observed" coefficients of lateral eddy diffusivity in stable water with computed values from equation (2). Atlantic Ocean mid-depths, circles; deep water, squares; California offing, x's; Long Island Sound, triangles; Georges Bank, dots.

physical parameters under consideration. However, none of the possibilities examined leads to realistic results. Conversely, the only reasonably accurate empirical proportionality that has been discovered (Fig. 2) cannot be incorporated with the other postulated parameters into a dimensionally correct expression for eddy conductivity.

DISCUSSION

Observations have shown that eddy diffusivity and conductivity in stable water are much smaller than eddy viscosity (Sverdrup, et al.,

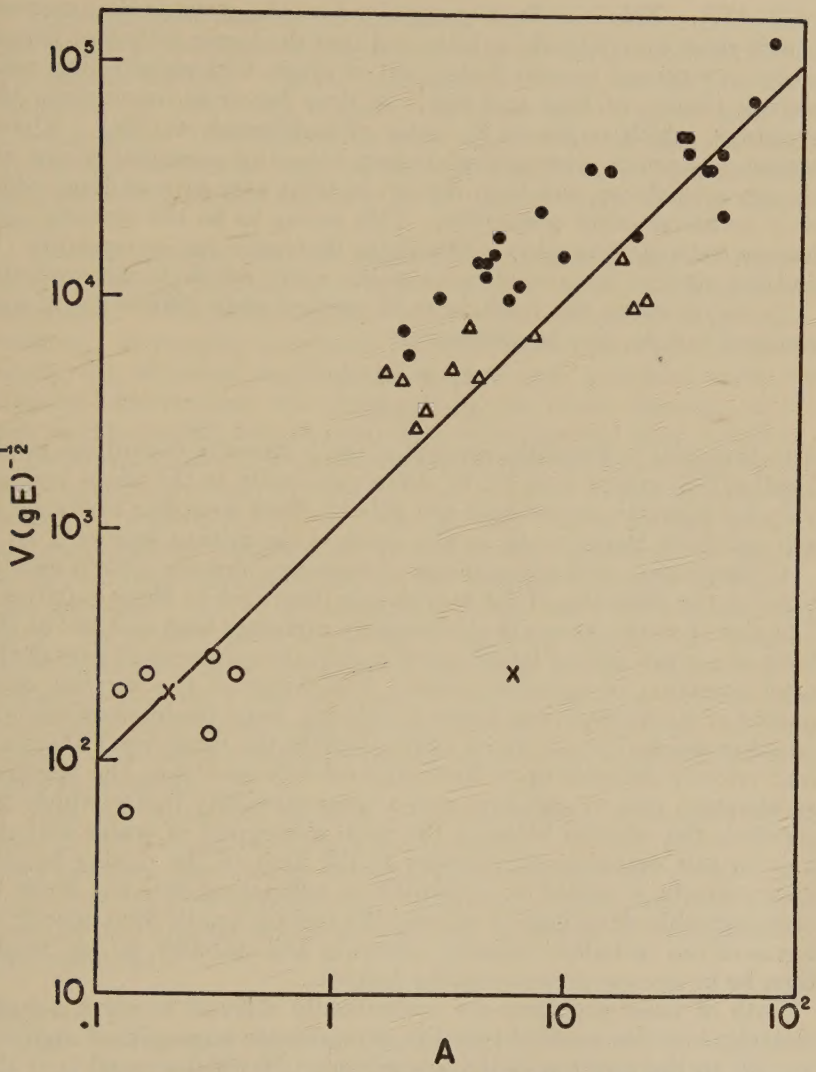

Figure 2. "Observed" coefficients of vertical eddy diffusivity and conductivity in stable water plotted against a function of the mean current speed and the stability. Symbols as in Fig. 1.

1942: 477). This is ordinarily explained on the grounds that momentum is given up relatively quickly and that the larger turbulent masses are then returned toward their point of origin without effecting complete exchange of heat and salt. A time factor is involved in the exchange, which is absent in water of indifferent stability. There, masses of various sizes and turbulent velocities presumably can exchange momentum and then remain in their new surroundings while they exchange other properties. This seems to be the essential difference between the eddy diffusivities that conform to equation (1) and the vertical conductivities in stable water which do not conform.

In stable water, the coefficients of vertical eddy diffusivity A_s and conductivity A_T may be written

$$A_s = \rho\, \bar{v}_s'\, l_s;$$
$$A_T = \rho\, \bar{v}_T'\, l_T.$$

This is similar to Prandtl's concept of eddy viscosity (Sverdrup, et al., 1942: 472–3) except that $\bar{v}_s'$, $\bar{v}_T'$ refer specifically to the mean vertical velocity of water masses that are able to effect complete exchange of salt and heat, respectively, at the limits of the mixing lengths l_s, l_T.

Consideration of the time factor of exchange imposes certain restrictions on the character of the movements described by these equations. The size of water masses that effectively exchange heat and salt at the limit of a given mixing length must constitute only a small part of the total spectrum of turbulent sizes. Variability of the vertical component of momentum, and hence of velocity, must be similarly limited. In other words, for any given mixing length, the mean vertical turbulent velocity depends upon the initial velocity spectrum, the negative acceleration due to stability, and a proportionality factor which determines the relation between the total movement of water and the heat or salt exchange that occurs at the limit of the mixing length. Alternatively, it might be expedient in comparing different kinds of oceanographic situations to express the mixing length dynamically in terms of the turbulent velocity spectrum and stability; v_s', v_T' would then be in essence proportionality factors.

Both of these problems are undoubtedly difficult to solve quantitatively, but it is possible that Fig. 2 represents an empirical approximation to the solution of the second one. It will be noted that the eddy coefficients are correlated with a quantity that has the dimensions of a length. The equation

$$A = \rho\, \bar{v}'\, v\, (g\, E)^{-\frac{1}{2}}$$

is empirically valid if $\bar{v}'$ is constant within the limits of observational error.

The coefficients cited in the APPENDIX were derived in order to apply the results to problems of distribution of plankton and minor chemical elements in the sea. This is mentioned to emphasize the fact that a knowledge of turbulence is becoming important in many aspects of oceanography. Therefore, it is unfortunate that the computation of turbulence coefficients from the observed distribution of oceanographic properties is laborious and subject to various errors of over-simplification.

There is obvious need for a method that will estimate turbulence with a degree of ease and accuracy comparable to the determination of advection. Theoretical studies thus far have failed to reveal such a method. It remains to be seen whether further theoretical work will verify the empirical equations that have been proposed above and whether observations will substantiate the initial assumption of a precise relationship between turbulent velocities and mean velocity.

However, from a practical standpoint, the results are more precise than might have been expected. Fig. 1, which compares coefficients computed from equations (1) and (2) with values derived from the observed salinity distribution, reveals a few differences of approximately an order of magnitude, but the average difference between the two methods in all of the estimates of lateral eddy coefficients is only 38%. At present there is no way of knowing whether the observed differences are due to observational errors or to imperfections in the equations that have been proposed. One can only say that no oceanographer has claimed, nor probably believed, that he could determine diffusion coefficients with an accuracy equal to the agreement between the two methods.

Thus the equations provide a possible means of simplifying the solution of various oceanographic problems. Computations of flux or of the local time change of oceanographic properties, which heretofore have required estimates of both advection and diffusion, can be transformed with the aid of the empirical equations into simple advection problems.

REFERENCES

McEWEN, G. F.
1950. A statistical model of instantaneous point and disk sources with applications to oceanographic observations. Trans. Amer. geophys. Un., *31:* 35–46.

PARR, A. E.
1936. On the probable relationship between vertical stability and lateral mixing processes. J. Cons. int. Explor. Mer, *11:* 308–313.

RILEY, G. A.
1951. Oxygen, phosphate, and nitrate in the Atlantic Ocean. Bull. Bingham oceanogr. Coll., *13* (1): 1–126.

1952. Hydrography of the Long Island and Block Island Sounds. Bull. Bingham oceanogr. Coll., *13* (3).

RILEY, G. A., HENRY STOMMEL, AND D. F. BUMPUS

1949. Quantitative ecology of the plankton of the western North Atlantic. Bull. Bingham oceanogr. Coll., *12* (3): 1–169.

SVERDRUP, H. U., M. W. JOHNSON, AND R. H. FLEMING

1942. The oceans. Prentice-Hall, Inc., New York. 1087 pp.

APPENDIX

General Explanation of Tabular Material

The sources of data are as follows:

1. Atlantic Ocean mid-depths and deep water (Riley, 1951). Coefficients of lateral (A_x) and vertical (A_z) eddy diffusivity are average values for a series of σ_t surfaces in mid-depths and level surfaces in deep water. They were computed statistically from the observed salinity distribution, the distance between points of observation x and z in cm; the current velocity is determined by dynamic computation. The current speeds v in cm/sec are the arithmetic average of the north-south and east-west velocities on each surface (for present purposes, a more accurate estimate would be the square root of the mean-squared vectorially-added velocities, but the general quality of the data hardly warrants this refinement). The tables herewith also include estimates of the dynamic molecular viscosity μ and the stability E, the latter approximated throughout as $10^{-3}\ \Delta\sigma_t/m$.

2. Hitherto unpublished estimates of eddy diffusion on two σ_t surfaces off the California Coast have been obtained by the same methods described above. This work was done while the writer was visiting Scripps Institution of Oceanography; grateful acknowledgment is made for the use of data from their Marine Life Research Program for this purpose.

3. Long Island Sound (Riley, 1952). Lateral eddy coefficients were determined from the distribution of salinity along the main axis of the Sound. Coefficients of vertical eddy conductivity were based on the vertical temperature gradients at a series of depths and on the seasonal rate of change of calories below each depth. v is the mean speed of the tidal current, computed from data on the height of the tide in the Sound and averaged for a complete tidal cycle. z is the total depth of water in cm.

4. Georges Bank (Riley, Stommel, and Bumpus, 1949). Coefficients of vertical eddy conductivity, as in Long Island Sound, are computed from average temperature-depth curves for the central part of the Bank. Only the upper 40 m are used for computation. Values

in deeper water appear to be reduced by proximity to the bottom. z is the mean depth of water; v is an approximation of the speed of the tidal current.

A

Water of slight stability (E is less than 50×10^{-8}). Eddy coefficients in Long Island Sound are averages for the whole autumn-winter period of minimum stability in five different areas; the Georges Bank estimate is an average for the upper 40 m.

	Depth	A_s	A_z	z	x	v	μ
Atlantic	1500	0.29	2.2×10^5	5×10^4	1×10^8	0.25	.017
Ocean	2000	0.21	2.3×10^5	5×10^4	1×10^8	0.29	.017
Deep	2500	0.34	6.4×10^5	5×10^4	1×10^8	0.35	.017
Water	3000	0.43	8.1×10^5	5×10^4	1×10^8	0.44	.017
	3500	0.51	8.6×10^5	5×10^4	1×10^8	0.59	.017
	4000	0.55	28.0×10^5	5×10^4	1×10^8	0.84	.017
Georges Bank*	—	140.0	—	7×10^3	—	100.0	.017
Long	—	53.0	—	37×10^2	—	60.0	.017
Island	—	21.0	—	27×10^2	—	55.0	.017
Sound	—	18.0	—	23×10^2	—	42.0	.017
	—	11.0	—	25×10^2	—	30.0	.017
	—	9.0	—	22×10^2	—	23.0	.017
	—	—	38×10^5	—	3.7×10^6	30.0	.017

* March

B

Lateral eddy coefficients in stable water. Long Island Sound coefficient and stability are averages for the spring-summer period.

	σ_t	A_x	x	v	μ	E
Atlantic Ocean	26.5	57.0×10^6	1×10^8	1.5	.012	290×10^{-8}
Mid-depths	26.7	18.0×10^6	1×10^8	1.2	.013	270×10^{-8}
	26.9	14.0×10^6	1×10^8	0.88	.013	210×10^{-8}
	27.1	10.5×10^6	1×10^8	0.78	.014	110×10^{-8}
	27.3	2.5×10^6	1×10^8	0.39	.015	90×10^{-8}
	27.5	2.4×10^6	1×10^8	0.15	.016	60×10^{-8}
Calif.	25.5	5.5×10^6	3.7×10^7	2.0	.014	1040×10^{-8}
Offing	27.0	0.5×10^6	3.7×10^7	0.8	.017	120×10^{-8}
Long Island Sound	—	4.9×10^6	3.7×10^6	30.0	.011	1020×10^{-8}

C

Vertical eddy coefficients in stable water. Values for Georges Bank are individual estimates for a series of depths. Long Island Sound coefficients are also individual estimates representing different depths and areas.

	σ_t	A_z	z	v	μ	E
Atlantic Ocean	26.5	0.30	70×10^2	1.5	.012	290×10^{-8}
Mid-depths	26.7	0.16	73×10^2	1.2	.013	270×10^{-8}
(Diffusivity)	26.9	0.12	95×10^2	0.88	.013	210×10^{-8}
	27.1	0.38	181×10^2	0.78	.014	110×10^{-8}
	27.3	0.31	225×10^2	0.39	.015	90×10^{-8}
	27.5	0.13	350×10^2	0.15	.016	60×10^{-8}
California Offing	25.5	0.20	48×10^2	2.0	.014	1040×10^{-8}
(Diffusivity)	27.0	6.0	420×10^2	0.8	.017	120×10^{-8}
Long Island Sound	—	7.2	—	60.0	.011	$720 \times 10^{-}$
(Conductivity)	—	19.0	—	60.0	.011	430×10^{-8}
	—	4.2	—	55.0	.011	1470×10^{-8}
	—	22.0	—	55.0	.011	300×10^{-8}
	—	17.0	—	55.0	.011	150×10^{-8}
	—	3.8	—	42.0	.011	310×10^{-8}
	—	3.2	—	30.0	.011	370×10^{-8}
	—	1.7	—	30.0	.011	400×10^{-8}
	—	2.0	—	23.0	.011	290×10^{-8}
	—	2.3	—	21.0	.011	630×10^{-8}
	—	2.5	—	21.0	.011	460×10^{-8}

	April $\mu = .017$		May $\mu = .016$		June $\mu = .014$	
	A_z	E	A_z	E	A_z	E
Georges Bank	45	60×10^{-8}	9.5	460×10^{-8}	35.0	720×10^{-8}
(Conductivity)	39	80×10^{-8}	6.3	780×10^{-8}	20.0	430×10^{-8}
$v = 100$ cm/sec	34	60×10^{-8}	4.5	660×10^{-8}	5.8	1120×10^{-8}
throughout	31	40×10^{-8}	4.5	540×10^{-8}	2.1	2980×10^{-8}
	32	40×10^{-8}	4.9	460×10^{-8}	2.0	1980×10^{-8}
	39	60×10^{-8}	7.7	200×10^{-8}	2.8	980×10^{-8}
	62	20×10^{-8}	12.5	80×10^{-8}	4.1	560×10^{-8}
	76	7×10^{-8}	15.0	80×10^{-8}	5.1	280×10^{-8}

DIURNAL FLUCTUATIONS IN BATHYTHERMOGRAMS[1,2]

By Philip Rudnick and John D. Cochrane

Scripps Institution of Oceanography
University of California

ABSTRACT

Series of bathythermograms extending over 12- to 90-day periods from two deep-sea stations in the northeastern Pacific fail to show more than indefinite or minor fluctuations of tidal period, in contrast with marked internal tides which have been observed in the Atlantic. Fluctuations of surface temperature were found to be about half random and half due to diurnal solar heating.

TIDAL PERIODICITY

The ocean stations maintained by the U. S. Coast Guard in the eastern Pacific have made it possible to obtain bathythermograms at frequent intervals over long periods of time. Three sets of such data, extending to a depth of 450 ft. (137 m), well below the thermocline in each case, have been investigated for fluctuations of tidal period. Table I and Fig. 1 describe the data. Spectra were obtained by computation of both periodograms (6) and correlograms (1), the latter with the help of the Marine Physical Laboratory correlator (7). The frequencies examined range from 0.3 cycles/day to the upper limit set by the spacing of the data, either 3 or 12 cycles/day.

In only one of the three cases (Series I) was a statistically significant result found. At 76 m depth, 14% of the total temperature variance (mean squared deviation, a rough measure of the "energy" of fluctuation) represented an approximately diurnal periodicity, the rest being distributed in a continuous spectrum, as shown in Fig. 2. Further numerical results are given in Table II. The other series, II and III, also gave some indications of diurnal or semidiurnal periods, but these were so small as to be of doubtful statistical significance.

TABLE I. Description of Data

Series designation	I	II	III
No. of bathythermograms	390	69	226
Interval between readings, in hours	1	4	4–4–16
Total interval, in days	16	12	91
Dates	19 June to 6 July 1949	12 August to 25 August 1948	1 July to 30 Sept. 1949
Position of stations	30° N, 140° W	30° N, 140° W	49° N, 148° W

[1] This work represents one of the results of research carried out under contract with the Bureau of Ships, Navy Department.

[2] Contribution No. 551 from the Scripps Institution of Oceanography.

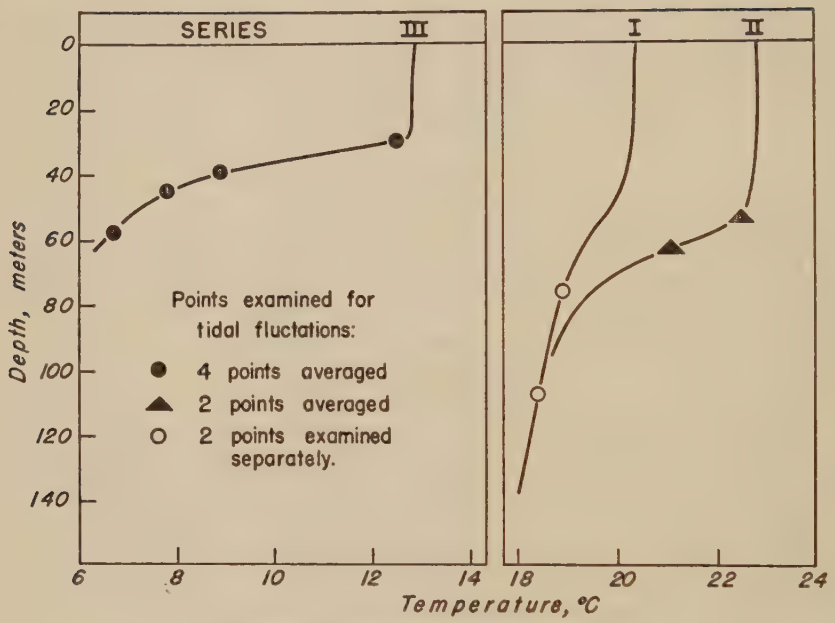

Figure 1. Mean bathythermograms.

In view of the absence of a semidiurnal component in Series I, it should be noted that these observations were taken within two degrees of an amphidromic point located by Harris (5) for the semidiurnal surface tide.

TABLE II. Results of Analysis of Series I

Depth, m	76	107
Total root-mean-square temperature fluctuation, in °C	0.26	0.22
Amplitude of component with 24.5 hour period, in °C	0.12	0.067
Corresponding presumable amplitude of vertical displacement, in m, using mean gradient from Fig. 1	5.4	4.7
Probability that random fluctuation of continuous spectrum (Fig. 2) would give an amplitude as great as that above, for one out of 18 periods investigated	0.003	0.3
Correlation coefficient with temperature at surface	0.09	0.19
Correlation coefficient between temperatures at depths of 76 and 107 m	0.48	

The diurnal component in Series I has been more closely examined for evidence of connection with tides. It was far from constant in magnitude during the 16 days of observations, being most prominent during the final four days, which included a neap-tide epoch. Minimum temperature (and minimum thermocline depth) in this period

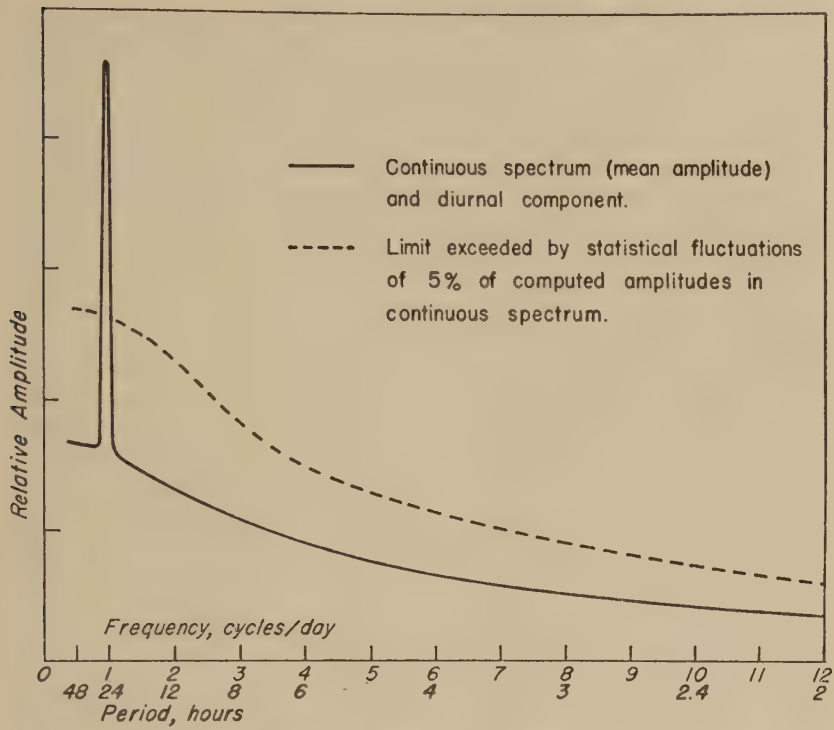
Continuous spectrum (mean amplitude) and diurnal component.
Limit exceeded by statistical fluctuations of 5% of computed amplitudes in continuous spectrum.
Relative Amplitude
Frequency, cycles/day
0 1 2 3 4 5 6 7 8 9 10 11 12
48 24 12 8 6 4 3 2.4 2
Period, hours

Figure 2. Spectrum for Series I.

occurred three hours later than the upward maximum of the local tide-generating force; the corresponding lag for the diurnal high-water on the surface is five hours in this area. During the remaining 12 days, which included a spring-tide epoch, the component was also traceable with about the same phase, but it was below the level of good statistical significance. Thus, the phase of the internal oscillation corresponds well with that of the surface tide, but the amplitude variation during the fortnight does not. It would seem that the connection with tides is at most a loose one.

Two alternative interpretations have been suggested. Since the observations were made at 30° latitude, where the period of motion in an inertial circle is diurnal, one might suppose that the rather abrupt appearance of distinguishable diurnal oscillations in the final four days of Series I was the result of some impulsive meteorological disturbance whose aftermath could easily involve intertial motion. However, the pertinent weather maps show nothing to support this

possibility. Another suggestion is that vertical convection might communicate the influence of diurnal solar heating from the surface to thermocline depth. This seems possible, since the indicated energy of internal wave motion is of the same order as the potential energy which might be developed over a 10-day period by diurnal increase in salinity or by lowering of temperature at the surface.

The most striking aspect of the results is the inconspicuousness of whatever diurnal or semidiurnal periods may exist, in marked contrast with some previous observations. For example, mere inspection of the temperature fluctuation curves of the METEOR anchor station 385 of 1938 (8) at 17° N and 46° W convinces one of prominent semidiurnal oscillations at thermocline depth (80–120 m). Of course, the present data are seriously limited in respect to depth and by the lack of salinity measurements, so that an internal tide at greater depths could have escaped observations. However, deep casts made in the same areas (e. g., CARNEGIE stations 125 at 52° N and 151° W in July 1929 and 146 at 32° N and 141° W in October 1929 [4]) show maximum density gradients at depths of the order of 100 m, which is therefore the region which will generally show the most prominent internal motions. Hence, it does not seem probable that the present data, even though limited to 137 m depth, would completely fail to reveal conditions, if they exist, similar to those found by the METEOR (8, 2) and the ATLANTIS (9, 10) in the Atlantic Ocean.

Defant has expressed the view that internal tides are as widespread as the surface tides (3). Their generation at ocean boundaries seems a very probable process; in contrast, the presence of internal tides in the deep ocean is difficult to understand dynamically. If one narrows the discussion accordingly by considering only deep-sea stations well removed from ocean borders (thus excluding, for example, the SNELLIUS observations in the East Indies) and those occupied for at least two days (for the sake of more conclusive statistics), it then appears that the number of observed cases in this category is not large. Our present result suggests that some limitations exist on the place and manner of occurrence of internal tidal oscillations, which, if established, may in turn contain clues as to their dynamic origin.

SURFACE TEMPERATURES

Unusually good agreement was found between the surface temperature readings from the Series I bathythermograms and the corresponding bucket temperatures. Therefore these have been given some further statistical study. It is assumed that correlation between the two sets of readings arises from true fluctuations of sea-surface tem-

perature and is limited by independent errors in the two methods of observations. This permits separate numerical evaluation of the true RMS (root-mean-square) fluctuations and of the RMS error in each type of observation.

Part of the variance of true temperatures arises from a diurnally periodic component with maximum occurring three hours past local noon; this is presumably due to solar heating. The remaining variance is considered a real random fluctuation, separated from instrumental errors. Table III contains the numerical results.

TABLE III. Statistics of Surface Temperature and Observational Errors for Series I

	RMS fluctuation, °C	*Proportion of total variance*
Bathythermogram errors	0.06	
Bucket temperature errors	0.10	
True surface temperature		
total	0.12	100%
random component	0.09	59%
diurnal component (amplitude 0.11 °C)	0.08	41%
Correlation coefficient between bucket and bathythermogram readings	0.68	

Acknowledgment of essential contributions to this study are due the Coast Guard weather-patrol ships and staff members of the Bathythermograph Division and of the Marine Physical Laboratory. Drs. Walter H. Munk and Gifford C. Ewing have contributed valued suggestions on interpretation of the data.

REFERENCES

(1) Bartlett, M. S.
1950. Periodogram analysis and continuous spectra. Biometrika, *37:* 1–16.

(2) Defant, Albert
1932. Die Gezeiten und inneren Gezeitenwellen des Atlantischen Ozeans. Wiss. Ergebn. dtsch. Atlant. Exped. Meteor., 1925–27, *7* (1).

(3) 1950. On the origin of internal tide waves in the open sea. J. Mar. Res., *9:* 111–119.

(4) Fleming, J. A., et al.
1944–1945. Observations and results in physical oceanography. Publ. Carneg. Instn., Sci. Res. Cruise VII Carnegie. Oceanography 1-A and 1-B. No. 545.

(5) Harris, R. A.
1904. Manual of tides. IVB. Cotidal lines for the world. Rep. U. S. Cst. geod. Surv. (1903–04), App. V: 313–400 + 41 charts.

(6) Kendall, M. G.
1946. Contributions to the study of oscillatory time series. Cambridge University Press.

(7) Rudnick, Philip
1950. A system for recording and analyzing random processes; in symposium on autocorrelation held at Woods Hole, Mass., 13–14 June 1949. Office of Naval Research, Washington, D. C.

(8) Schubert, O. von
1944. Ergebnisse der Strommessungen und der ozeanografischen Serienmessungen auf den beiden Ankerstationen der zweiten Teilfahrt, Deutsche Nordatlantische Expedition, 1937 und 1939. Ann. Hydrogr. Berl., Januar Beiheft.

(9) Seiwell, H. R.
1937. Short period vertical oscillations in the western basin of the North Atlantic. Pap. phys. Oceanogr., *5* (2): 1–44.

(10) 1942. An analysis of vertical oscillations in the southern North Atlantic. Proc. Amer. phil. Soc., *85:* 136–158.

MIDSUMMER RENEWAL OF OXYGEN WITHIN THE HYPOLIMNION[1]

By

R. A. BRYSON AND V. E. SUOMI

Department of Meteorology
University of Wisconsin

ABSTRACT

Following a very rainy period in the summer of 1950, oxygen temporarily reappeared in the hypolimnion of Lake Mendota. It is postulated that the oxygen was introduced there by density currents of cold, silt laden, well oxygenated runoff water.

In the well known annual march of stratification in our northern lakes, the content of dissolved oxygen within the hypolimnion approaches nil soon after the onset of summer stratification (2), and the division between well oxygenated water above the thermocline and less oxygenated water below normally persists until the fall overturn. This reduction of oxygen in the hypolimnion occurred in Lake Mendota in the early part of the summer of 1950 as usual. However, in mid-July of 1950 there occurred in this region the rainiest five-day period recorded in the 85-year history of the Madison Weather Bureau Office, about 10 inches of rain falling on the Mendota watershed. Immediately following the period of rain, the Lake rose about 15 inches, and unusually high values of dissolved oxygen were measured at several levels within the hypolimnion.

Dissolved Oxygen (ppm)

Depth (m)	*July 11*	*July 17–18*	*July 21*	*July 26*	*August 10*
0–8	5	8.2	5.7–7.3	6.8–8.6	12.0–12.5
14–20	0–2.2	0.4–9.5	0.2–7.7	0–3.3	0– 0.5

Within a week after the rain, the oxygen values below the thermocline had returned to their normal low values. Evidently the abnormal appearance of oxygen in the lower strata of Lake Mendota was associated with the excessive rainfall. It is the purpose of this paper

[1] This paper represents a portion of the cooperative efforts of biologists, sanitary engineers, and meteorologists working under the auspices of the University of Wisconsin Lakes and Streams Investigations Committee. The authors wish to acknowledge gratefully the help of the score or so who contributed their efforts.

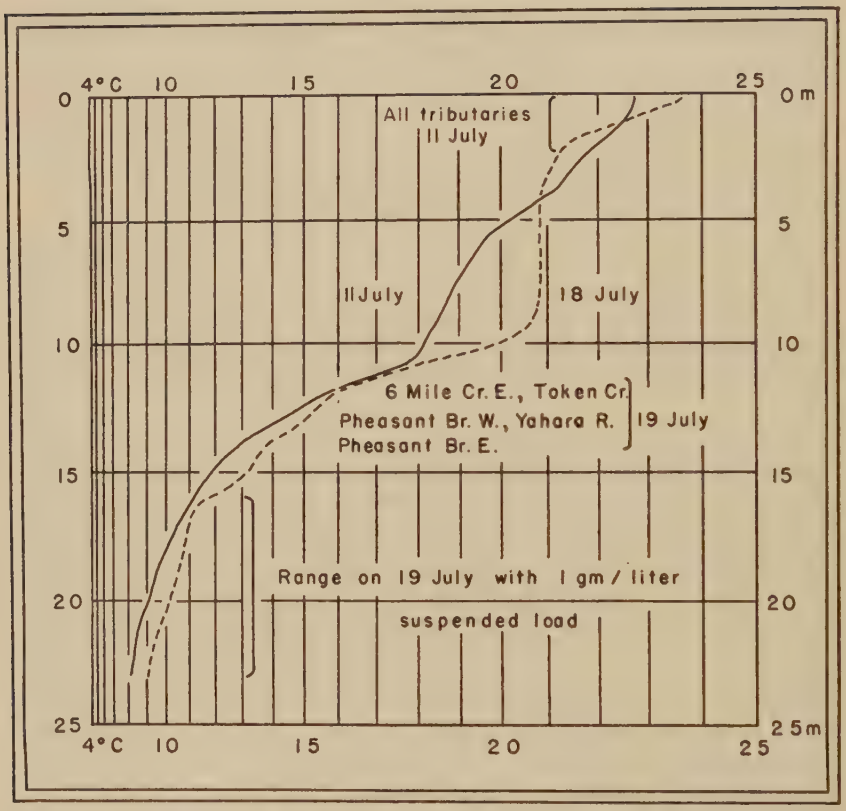

Figure 1. Change in level at which tributary waters spread due to cooling and addition of suspended load following heavy rains. The horizontal ordinate is linear with respect to density.

to suggest a mechanism whereby this renewal of oxygen could be accomplished.

Byers, et al. (1) have shown that rain temperatures in thunderstorms range from just above freezing to about the wet bulb temperature of the air. During the period in question, the wet bulb temperature of the air varied between 55° and 60° F and water temperatures within the hypolimnion ranged from 50° to 60° F. Thus the range in density of the rainwater was well below that found in the epilimnion, where temperatures of 70° to 72° F were observed. Even if the rainwater mixed with a considerable amount of this warm upper layer it might still be cold enough to sink below the epilimnion, carrying oxygen with it.

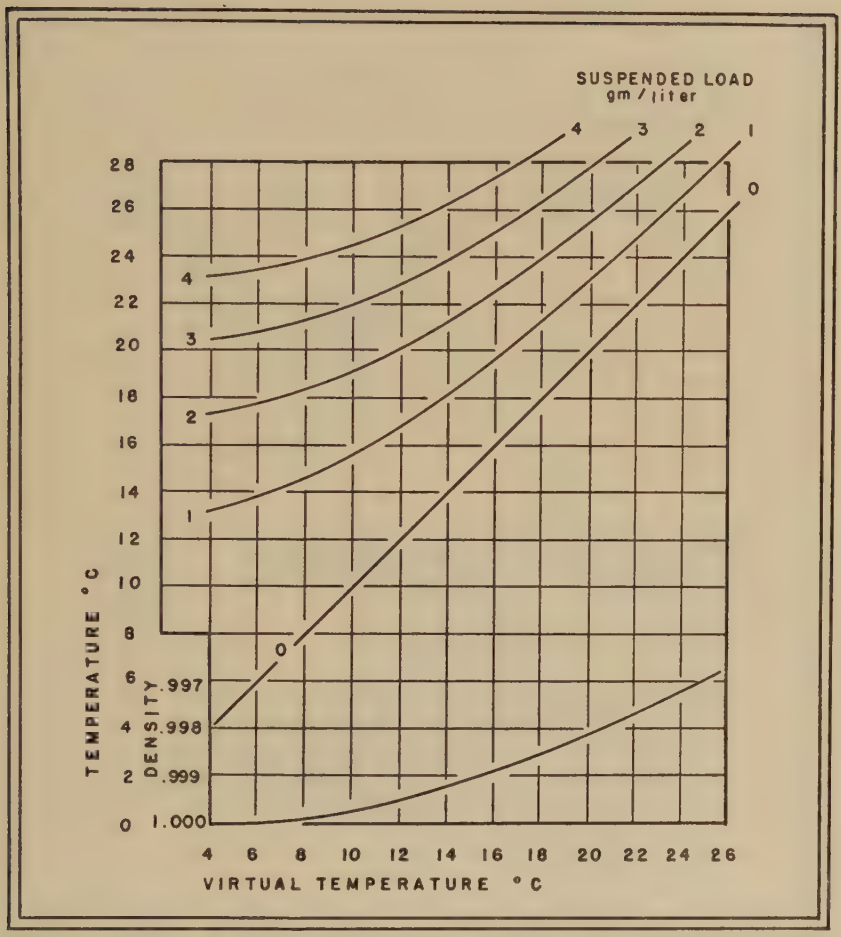

Figure 2. Graph of suspended load as a function of temperature and density. Note that a virtual temperature may be used to express the increase in density due to the suspensoids.

There are several reasons to believe that this was not the case. Before the rain, the water leaving Lake Mendota through the Yahara Locks had an alkalinity of 154 ppm., after the rain only 120 ppm. This outflow, from the upper meter, indicates that the rain actually mixed with and diluted the upper epilimnion.

On the other hand, the streams tributary to Lake Mendota provided a mechanism for carrying dissolved oxygen into the hypolimnion. Prior to the rain, the water from the tributaries, warmer than the

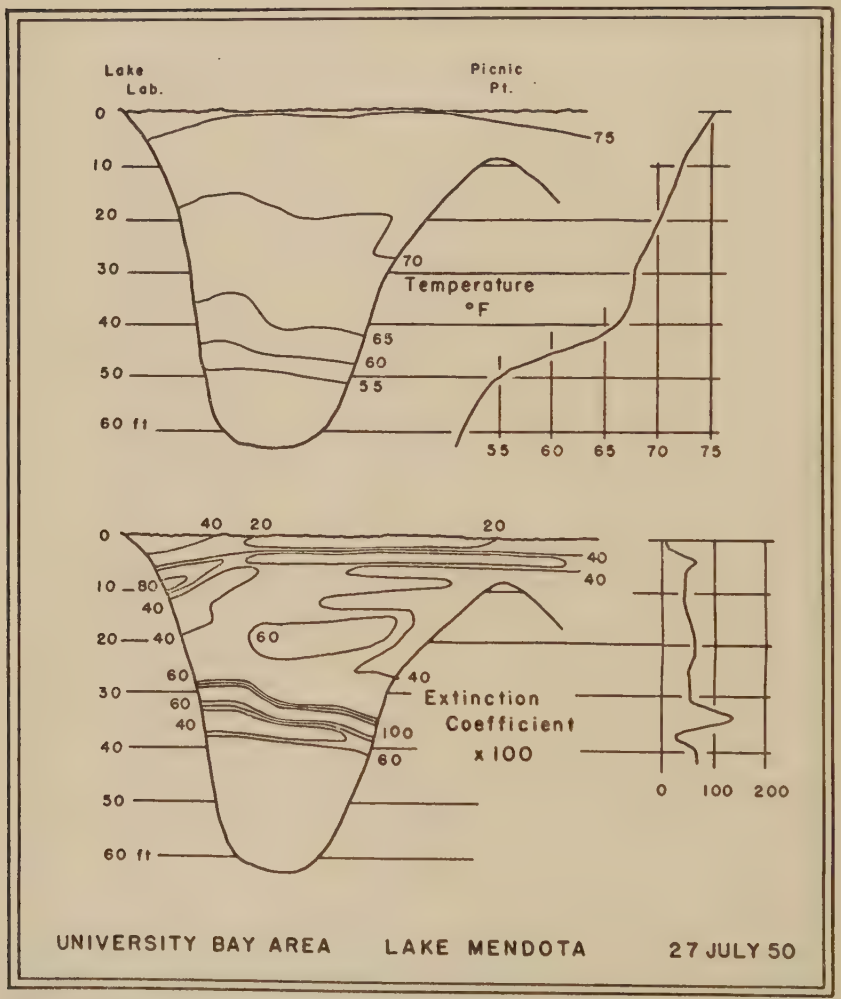

Figure 3. Distribution of temperature and extinction coefficient along a section across the mouth of University Bay, Lake Mendota. Length of section approximately one mile.

warmest lake water, would be expected to spread on the surface. After the rain, the muddy and well-aerated runoff water was cold. The density level in the Lake corresponding to this temperature was within the thermocline and upper hypolimnion (Fig. 1). However, as may be seen from Fig. 2, the effect of suspended load is to increase the density. If a conservative value of suspended load is assumed, a

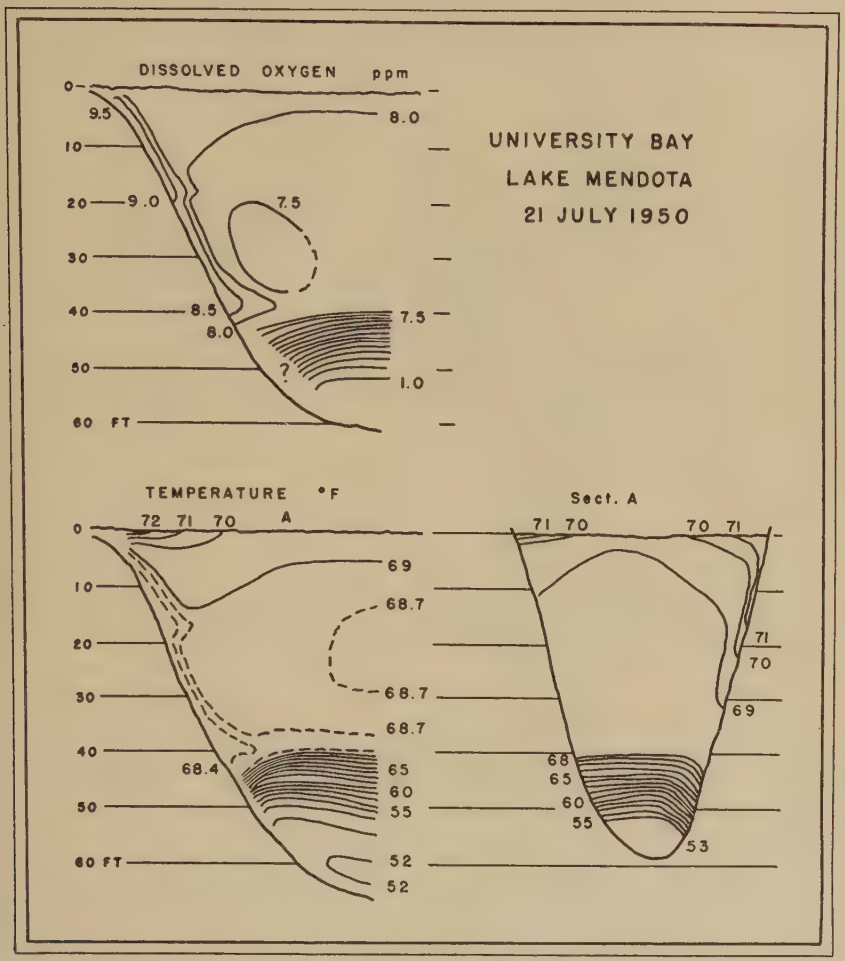

Figure 4. Longitudinal profiles of dissolved oxygen and temperature through University Bay. Section A is a transverse profile at the point marked "A."

density for the runoff water corresponding to a much lower temperature is obtained from the graph. Referring again to Fig. 1, it is clear that this tributary inflow could have descended to the lower part of the hypolimnion to spread.

Since each tributary has its own particular combination of temperature and suspended load, each should find a different density surface over which to spread. Stratification of turbidity results (Fig. 3).

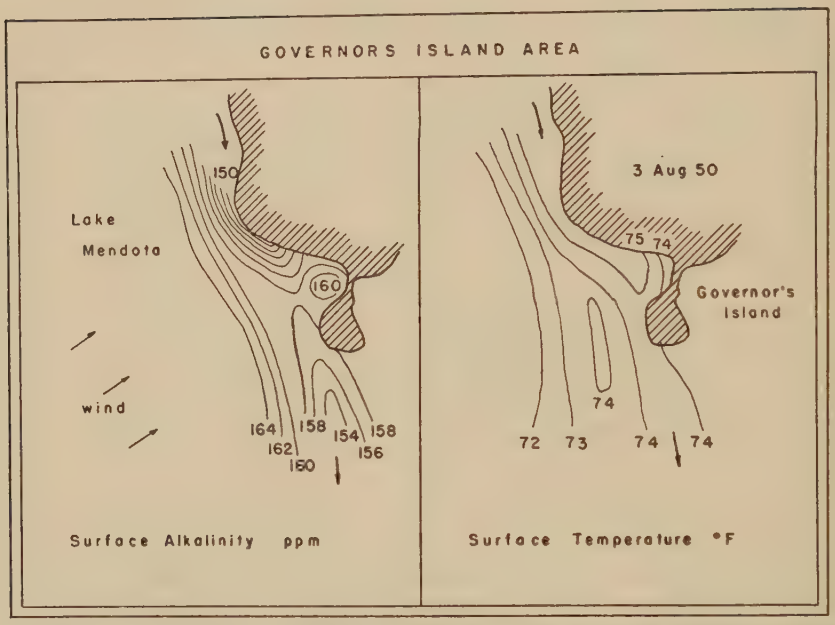

Figure 5. Distribution of alkalinity and temperature in the surface water around Governor's Island, north shore of Lake Mendota, August 3, 1950.

With sufficient suspended load, the density may be raised enough to reverse the normal downward decrease of temperature. Fig. 4 illustrates such a case; the muddy water which descended the slope at the head of University Bay was responsible for both the high oxygen values at depth and the downward increase of temperature at the bottom.

An example of a sediment-laden stream entering the Lake is shown in Figs. 5 and 6. The Yahara River stream entering the Lake is clearly delineated by the alkalinity, temperature, and turbidity distributions at and near the surface, but a cross section shows that the main body of muddy water is flowing along the bottom.

During and immediately following the periods of extremely heavy rain, many of the streams tributary to Lake Mendota ran at flood stage, with cold water carrying a large load of suspended sediment and abnormal amounts of dissolved oxygen. These tributaries provide a mechanism for occasional mid-summer renewal of hypolimnetic oxygen.

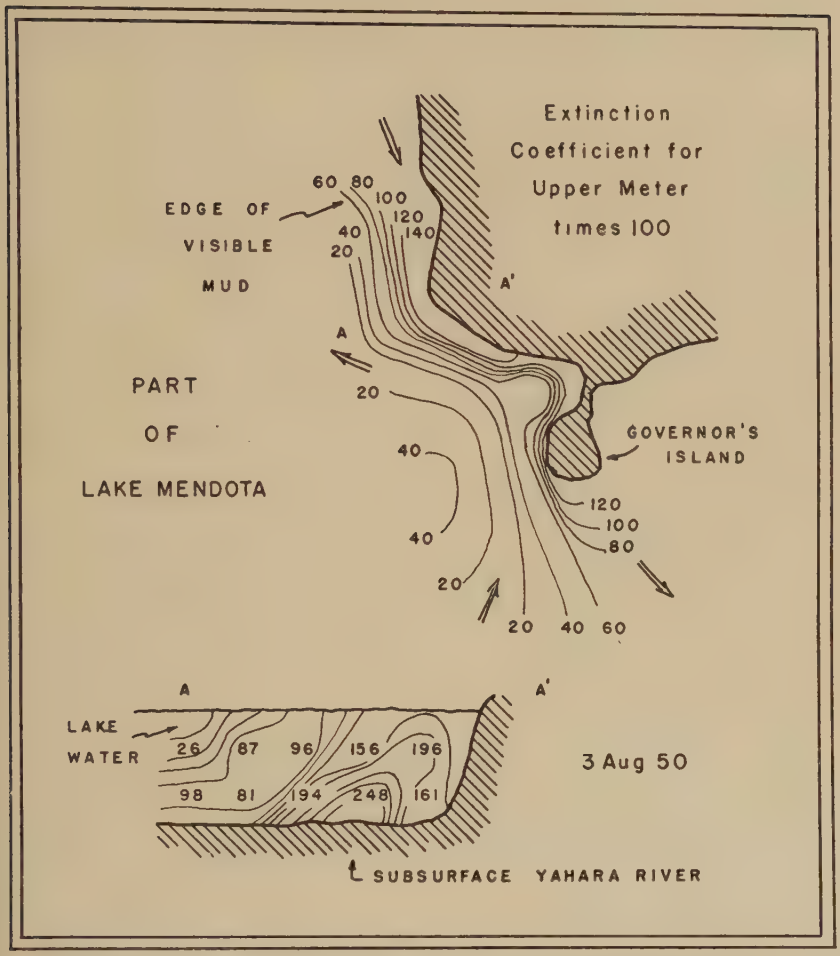

Figure 6. Extinction coefficient in the same area and at the same time as in Fig. 5.

REFERENCES

(1) Byers, H. R., Harry Moses, and P. J. Harney
1949. Measurement of rain temperature. J. Meteorol., *6:* 51–55.
(2) Welch, P. S.
1935. Limnology. McGraw-Hill, New York. 471 pp.

NOTE TO AUTHORS

The pages of this Journal are open to all contributors, and manuscripts may be submitted to any member of the Board of Editors or to the Managing Editor. Publication will be made only in the English language, but copy may be submitted for consideration in other languages. Aid in translation of accepted manuscripts will be given when possible.

All manuscripts should be typewritten with double space, carefully corrected, with footnotes continuously numbered and set off in the body of the text. If an article is to appear as a contribution from another institution, it should be approved by that institution before submission; contribution numbers of such institutions are submitted with the return of galley proofs.

The author (or authors) of each article will receive a total of 50 free reprints (with covers). Additional reprints with or without covers may be ordered by authors or institutions at the cost of printing. Orders for reprints are given when galley proofs are returned.

Both galley and page proofs are submitted to the senior author of each article.

Tables should be prepared for clarity and as near as practicable to Journal pattern, without ruled lines if possible. Table footnotes should appear at the base of each Table.

Figures are required for consideration simultaneously with the manuscript. Original figures of inconvenient size for forwarding may be represented by photographs on first submission of a manuscript; originals are required for line cuts. Illustrations requiring half-tone reproduction should be avoided as far as possible, but they will be accepted when necessary. *It is very important that the proper size of characters and symbols should be used for the lettering of illustrations.* A diagram for determining suitable heights of letters for original drawings that will be reduced, together with an explanatory note, has been published in this Journal, Vol. III, No. 1: 89–91; reprints will be submitted to authors on request. Mechanical lettering is recommended. If facilities for lettering are not available to the author, it is preferable to receive illustrations with lettering indicated in pencil only, to be finished in ink in the editorial office. All illustrations will be subject to at least a minimum reduction at the rate of 1 : 1.2. Drawings which have been heavily patched and pasted must *not be rolled* for mailing. The legends for the figures, identified by corresponding figure numbers on the illustrations also, should be typed in sequence at the end of the manuscript.

A complete and *accurate list of references* in alphabetical order at the end of each article is essential. References may be indicated in the text by author's name (or names) and *year of publication*, or by italicized numbers which correspond to the numbers given in sequence in the list of references. Each reference should include the last name followed by at least two initials or first name; co-authors should be listed with the initials or first name preceding the last name. The date should be indented on the line below, followed by a complete title *verbatim* in lower case except for the first word. Publisher, city and pagination should be indicated for books; for periodicals (abbreviations from "World List of Scientific Periodicals," Oxford, as far as possible), the volume (italicized), number and pagination should be given.

Additional information regarding the preparation of manuscripts will be furnished by the Managing Editor.

TABLE OF CONTENTS

Page

1 **G. A. Riley:** Parameters of turbulence in the sea 247

2. **Philip Rudnick and J. D. Cochrane:** Diurnal fluctuations in bathythermograms 257

3. **R. A. Bryson and V. E. Suomi:** Midsummer renewal of oxygen within the hypolimnion 263

SEARS FOUNDATION FOR MARINE RESEARCH
BINGHAM OCEANOGRAPHIC LABORATORY, YALE UNIVERSITY

JOURNAL OF MARINE RESEARCH

VOLUME XI, 1952

NUMBERS 1–3

ISSUED 1952, NEW HAVEN, CONN., U. S. A.

JOURNAL OF MARINE RESEARCH

PUBLISHED BY

SEARS FOUNDATION FOR MARINE RESEARCH

BINGHAM OCEANOGRAPHIC LABORATORY, YALE UNIVERSITY

BOARD OF EDITORS

Chairman: A. E. PARR
American Museum of Natural History, New York

C. O'D. ISELIN
Woods Hole Oceanographic Institution

H. U. SVERDRUP
Norsk Polarinstitutt, Oslo, Norway

DANIEL MERRIMAN
Bingham Oceanographic Laboratory

C.-G. ROSSBY
Institut für Meteorologi, Stockholm, Sweden

THOMAS G. THOMPSON
Oceanographic Laboratories, University of Washington

MANAGING EDITOR

YNGVE H. OLSEN
Bingham Oceanographic Laboratory
Yale University
New Haven, Conn.